Post Trade Solutions

Open-Source Risk Engine & Risk Analytics Lab Masterclass

LSEG

Welcome Keynote / Overview

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Co-Head Quantitative Services

James Mac Hale

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Post Trade Solutions

Update on Models:

SVI Model

Alex Chow

Senior Consultant

Post Trade Solutions

Update on Models:

ORE Local Vol Exposure Simulation

Peter Caspers

Quant Development Lead

Post Trade Solutions

Local Vol

Local Vol Model:

$$dS(t) = S(t)\big(r(t) - q(t)\big)dt + S(t)\sigma\big(t, S(t)\big)dW^S$$

Goal: Calibrate $\sigma\big(t, S(t)\big)$ to market call prices $\mathrm{C}(\mathrm{T}, \mathrm{K})$.

Fokker-Planck Forward PDE for probability density $\mathrm{p}(\mathrm{S}, \mathrm{t})$ of $\mathrm{S}(\mathrm{t})$:

$$\frac{\partial}{\partial t}p(S,t) = -\frac{\partial}{\partial S}p(S,t)S(t)\big(r(t) - q(t)\big) + \frac{\partial^2}{\partial S^2}p(S,t)S^2(t)\frac{\sigma^2\big(t,S(t)\big)}{2}$$

Call Price:

$$C(T,K) = P(0,T)\int_K^\infty (S-K)p(S,T)dS$$

Combining Fokker-Planck with Call Price Integral formula:

Local Vol

Combining Fokker-Planck with the Call Price Integral formula:

$$\frac{\partial C(T,K)}{\partial T} = -K\big(r(\mathrm{T}) - q(\mathrm{T})\big)\frac{\partial C(T,K)}{\partial K} - q(T)C(T,K) + \frac{1}{2}\sigma(\mathrm{T},K)^2 K^2 \frac{\partial^2 C(T,K)}{\partial K^2}$$

Solve for local volatility:

$$\sigma(\mathrm{T},K)^2 = \frac{\frac{\partial C(T,K)}{\partial T} + K\big(r(\mathrm{T}) - q(\mathrm{T})\big)\frac{\partial C(T,K)}{\partial K} + q(T)C(T,K)}{\frac{1}{2}K^2\frac{\partial^2 C(T,K)}{\partial K^2}}$$

”Dupire Forward Equation”.

Local Vol Demo (deterministic rates, FX Black Scholes)

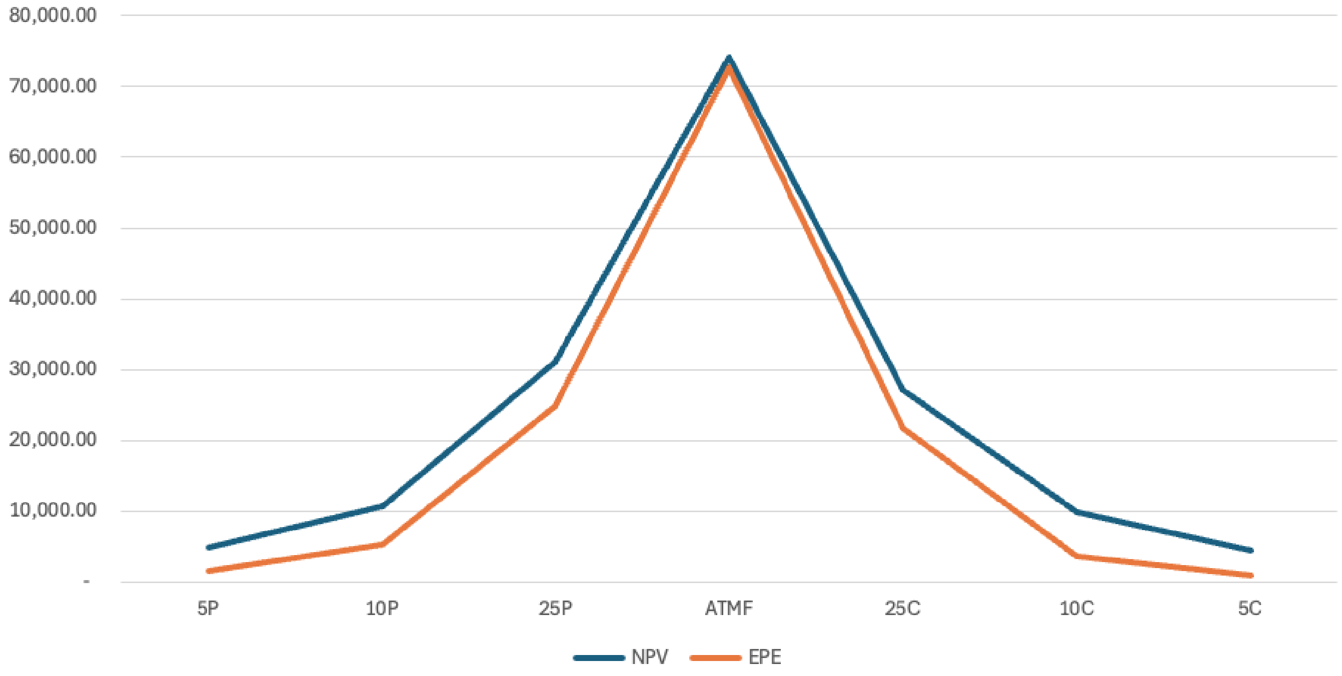
- T0 NPV of FX Options 5Y EUR/USD as of 2025-09-30 vs

  EPE at 5Y from Exposure Simulation of the underlying FX Forwards

- Exposure Simulation Model is

  - FX Black-Scholes calibrated on ATM FX Options 1Y, 2Y, 3Y, 4Y, 5Y

  - deterministic interest rates



Local Vol Demo (deterministic rates, FX Local Vol)

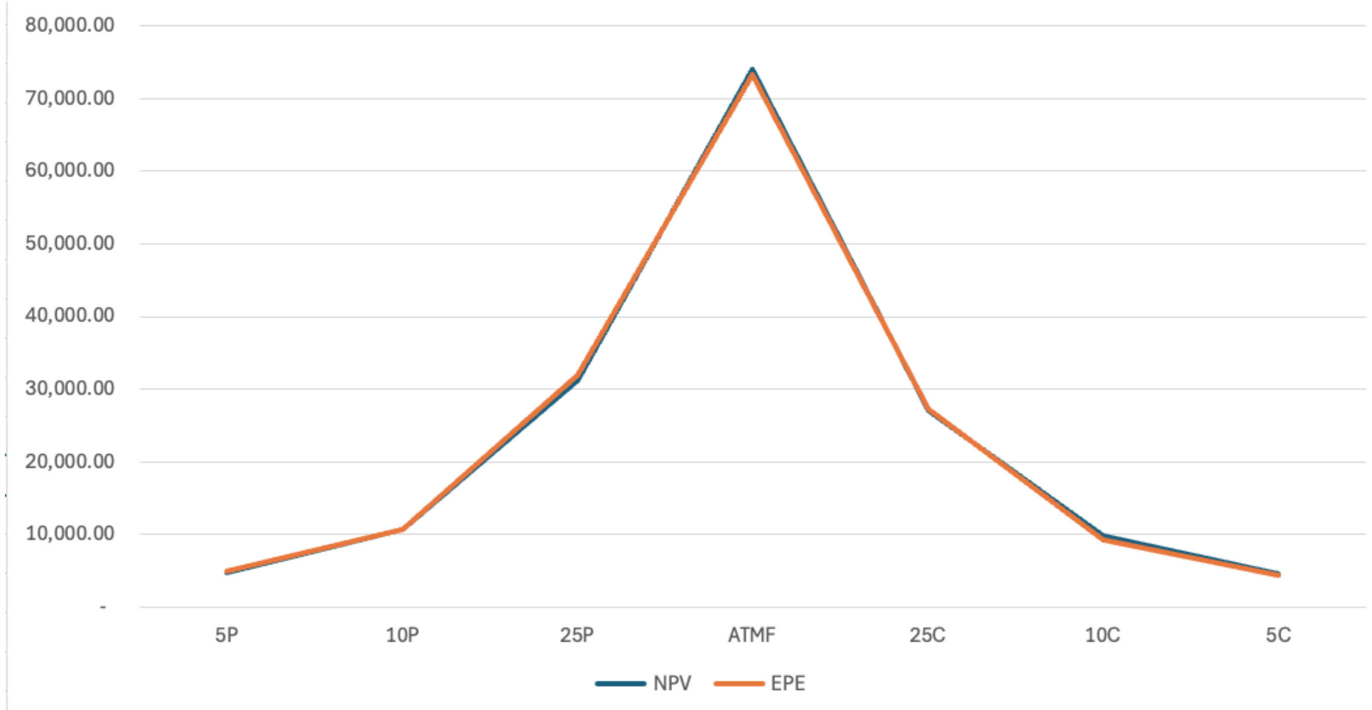
- T0 NPV of FX Options 5Y EUR/USD as of 2025-09-30 vs

  EPE at 5Y from Exposure Simulation of the underlying FX Forwards

- Exposure Simulation Model is

  - FX Local Vol calibrated on ATM FX Options 1Y, 2Y, 3Y, 4Y, 5Y

  - deterministic interest rates



Local Vol Demo (stochastic rates, FX Local Vol w/o stochrate corr)

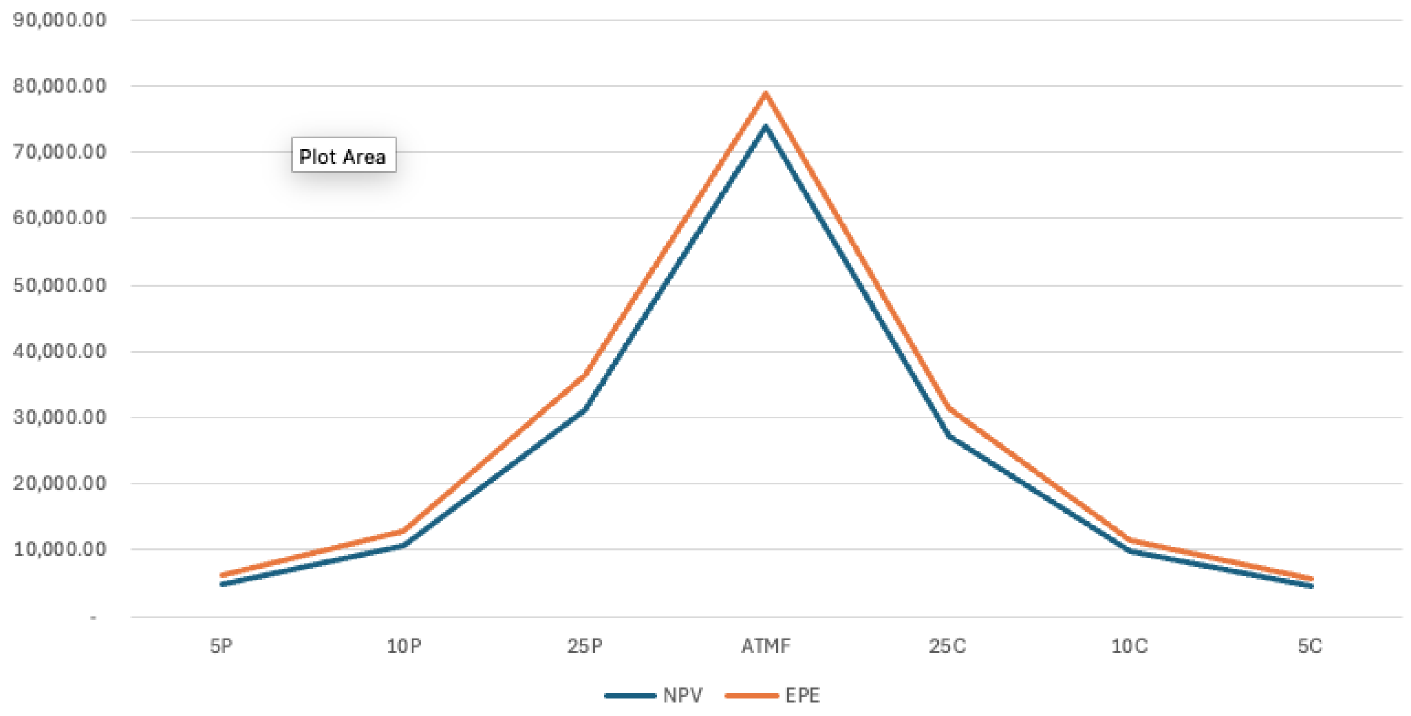
- T0 NPV of FX Options 5Y EUR/USD as of 2025-09-30 vs

  EPE at 5Y from Exposure Simulation of the underlying FX Forwards

- Exposure Simulation Model is

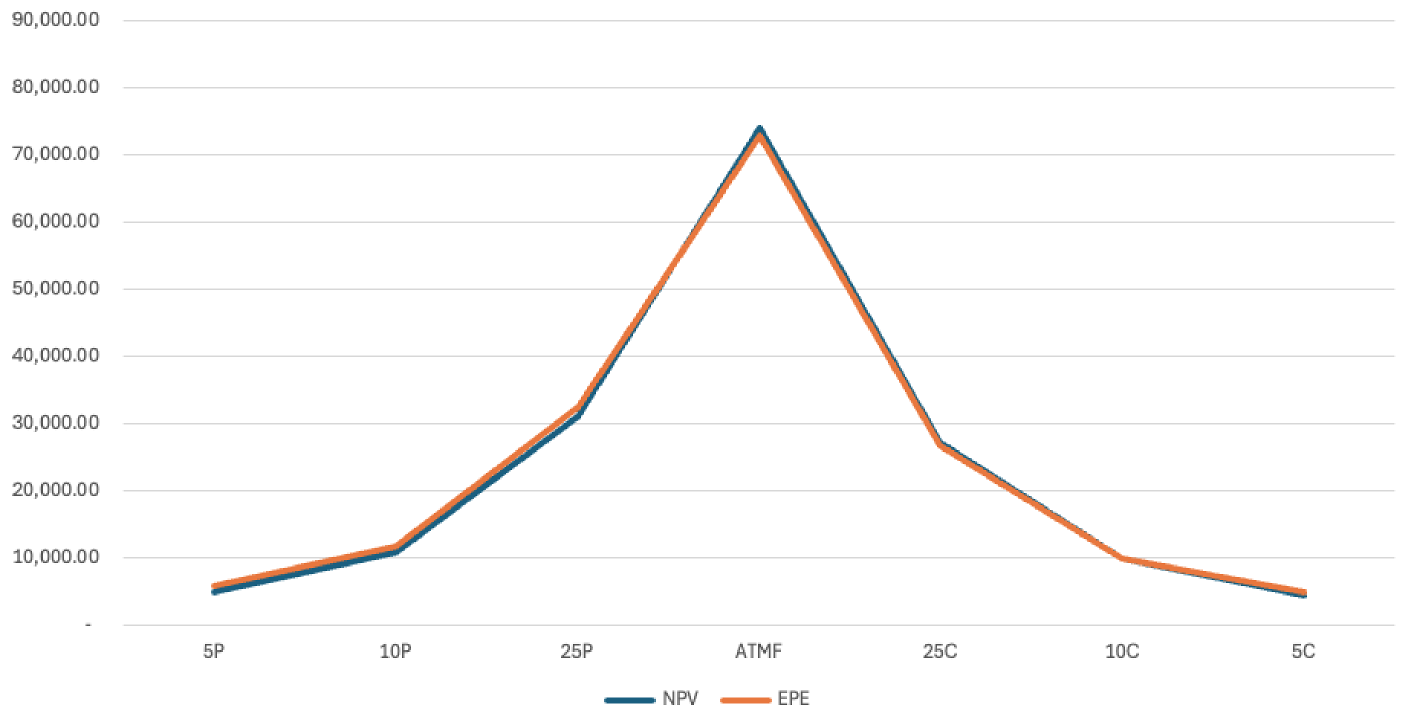
  - FX Local Vol calibrated on ATM FX Options 1Y, 2Y, 3Y, 4Y, 5Y

  - LGM1F calibrated to 10Y Coterminal Strip, correlation IR/FX 0.3

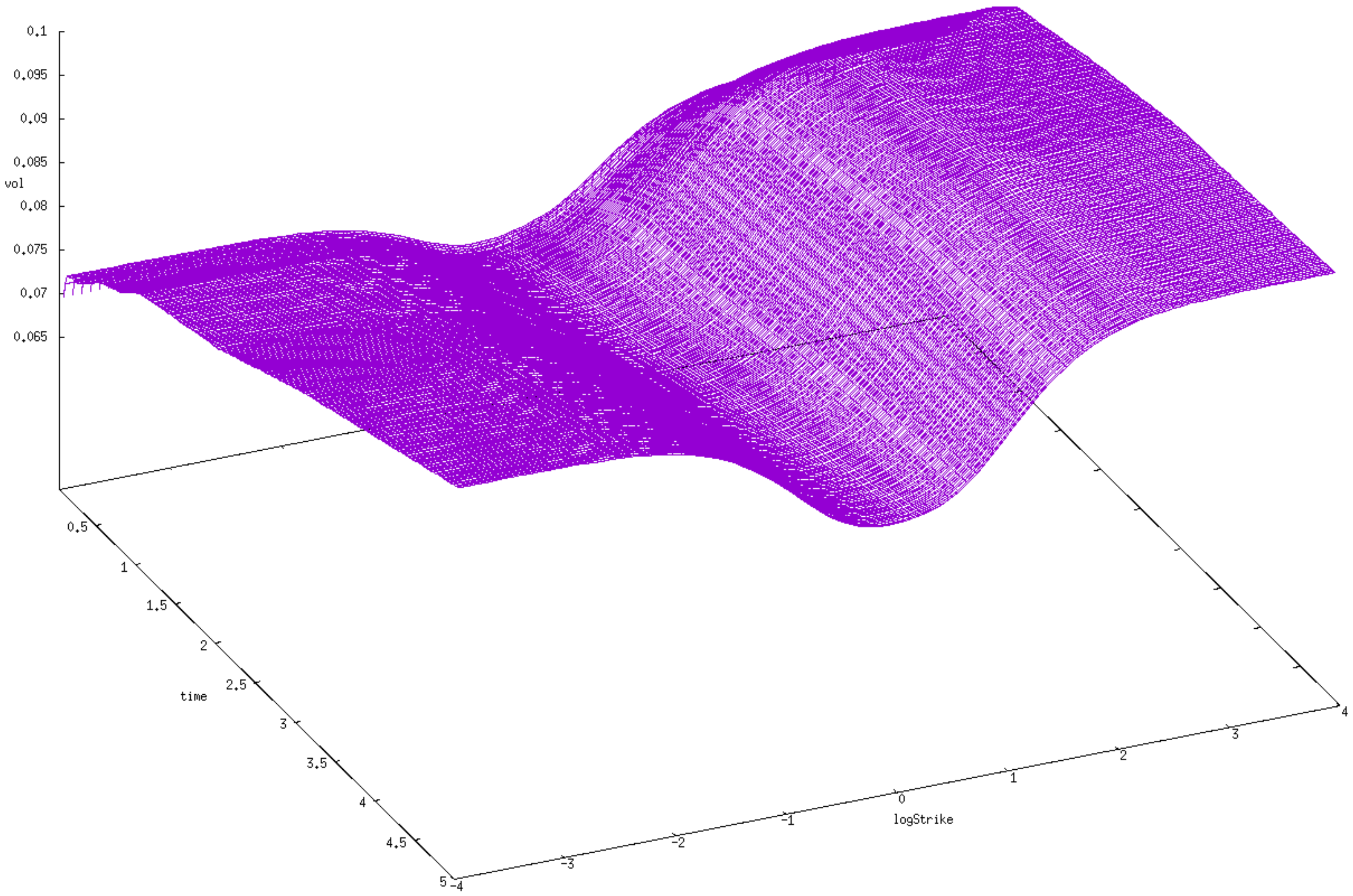


Local Vol Demo (stochastic rates, FX Local Vol w/ stochrate corr)

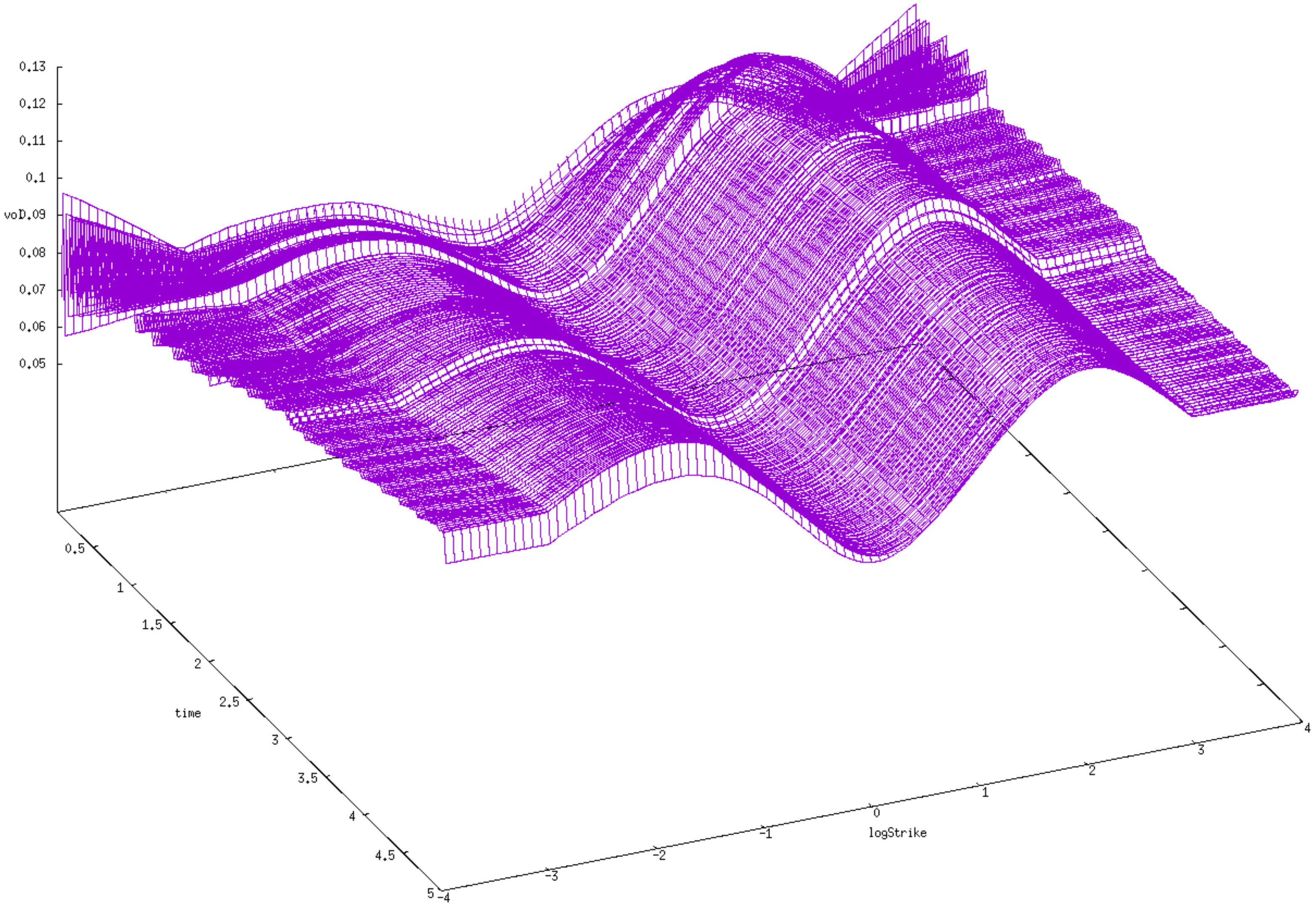
- T0 NPV of FX Options 5Y EUR/USD as of 2025-09-30 vs
  EPE at 5Y from Exposure Simulation of the underlying FX Forwards
- Exposure Simulation Model is
  - FX Local Vol calibrated on ATM FX Options 1Y, 2Y, 3Y, 4Y, 5Y
  - LGM1F calibrated to 10Y Coterminal Strip, correlation IR/FX 0.3



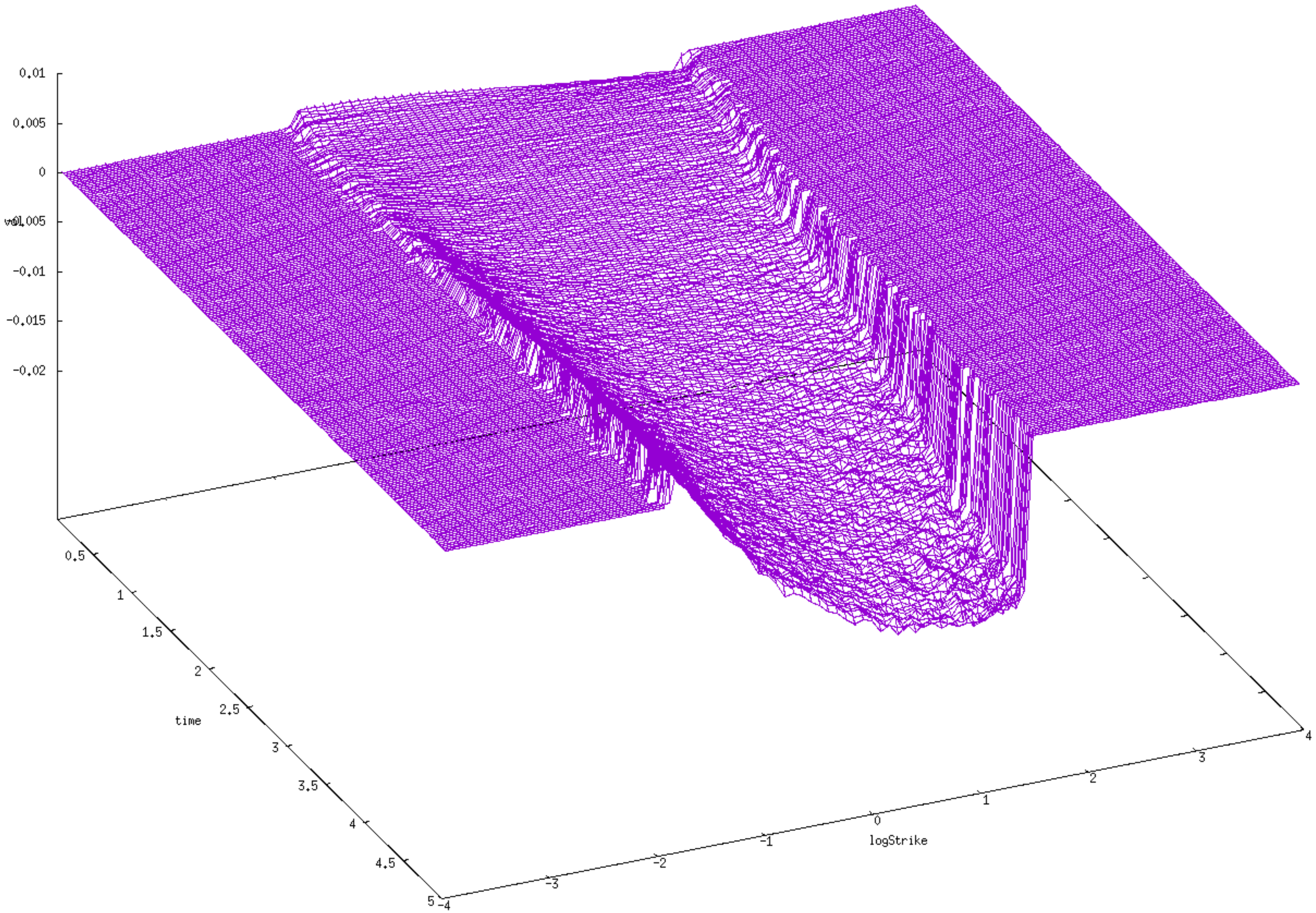
Local Vol Demo: Black Volatility



Local Vol Demo: Local Volatility (Andreasen – Huge Construction)



Local Vol Demo: Correction to Local Vol due to stochrates



Stochastic rates correction for Local Vol

Local vol for deterministic rates:

$$\sigma(T,K)^2 = \frac{\dfrac{\partial C(T,K)}{\partial T} + K\left(f_r(0,T) - f_q(0,T)\right)\dfrac{\partial C(T,K)}{\partial K} + f_q(0,T)C(T,K)}{\dfrac{1}{2}K^2\dfrac{\partial^2 C(T,K)}{\partial K^2}}$$

Stochastic rates formula for local vol can be derived using the multidimensional version of Fokker-Planck Forward PDE applied to joint SDE for S(t) and an IR model (e.g. Hull-White):

$$\sigma^*(T,K)^2 = \frac{\dfrac{\partial C(T,K)}{\partial T} - K\mathbf{E}\left(\left(r(T) - q(T)\right)D(T)1_{S^*>K}\right) + \mathbf{E}(q(T)D(T)(S^* - K)^+)}{\dfrac{1}{2}K^2\dfrac{\partial^2 C(T,K)}{\partial K^2}}$$

Stochastic rates correction for Local Vol

Correction Formula:

Let $S(t)$, $S^*(t)$ be the asset prices evolved without and with stochastic rates, i.e.,

$$dS(t) = S(t)\left(f_r(0,t) - f_q(0,t)\right) dt + \sigma\big(t, S(t)\big) dW(t)$$

$$dS^*(t) = S^*(t)\big(r(t) - q(t)\big)dt + \sigma^*\big(t, S^*(t)\big)dW(t)$$

Then

$$\sigma^*(\mathrm{T},K)^2 - \sigma(\mathrm{T},K)^2 = \frac{-K\left[E\left(\big(r(T)-q(T)\big)D(T)1_{S^*>K}\right) - E\left(\left(f_r(0,T)-f_q(0,T)\right)P(0,T)1_{S>K}\right)\right] + E\big(q(T)D(T)(S^*-K)^+\big) - E\big(f_q(0,T)P(0,T)(S-K)^+\big)}{\frac{1}{2}K^2E\big(D(T)\delta(S^*-K)\big)}$$

Monte Carlo Simulation of Correction $\sigma^*(t,K)^2 - \sigma(t,K)^2$

1 define time discretization $t_0, t_1, \ldots, t_n$:

2 for $\mathrm{i} = 0, \ldots, \mathrm{n} - 1$:

3 evolve $S^*(t), r(t), q(t)$ (stoch rates) from $t_i \to t_{i+1}$ on n MC paths
$S(t)$ (det rates) from $t_i \to t_{i+1}$ on n MC paths

4 estimate $E\left(\left(r(T) - q(T)\right)D(T)1_{S^*>K}\right)$ and $E\left(\left(f_r(0,T) - f_q(0,T)\right)P(0,T)1_{S>K}\right)$
$E(q(T)D(T)(S^* - K)^+)$ *and* $E\left(f_q(0,T)P(0,T)(S - K)^+\right)$

5 estimate $E\left(D(T)\delta(S^* - K)\right)$

6 calculate $\sigma^*(\mathrm{T},K)^2 - \sigma(\mathrm{T},K)^2$

7 iterate through 3 – 6 with the updated local vol correction until convergence

Update on Models:

Two-factor Commodity Model for Exposure Simulation in ORE

Sarp Kaya Acar

Principal Consultant

Post Trade Solutions

Commodity Model Evolution Overview

- ***Legacy stage***:
  - The initial one-factor model baseline **without seasonality**
  - **Cross correlation** between COM and other asset classes are **not supported**

- ***Current Master Branch***:
  - One-factor model **supports seasonality**
  - **Cross correlation** between COM and other asset classes are **supported**

- ***Work in Progress*****:** Expands model structure to a N-factor model with seasonality and a calibration strategy for a **two-factor setup with seasonality**. Andersen[2008]: "Markov Models for Commodity Futures: Theory and Practice"

- ***Next step*****:** Plans the calibration of N-factor model as the next near/mid-term deliverable depending on project priorities

One-Factor Model

Instantaneous forward rate $F(t,T)$ is modeled in HJM framework

$$dF(t,T) = F(t,T)\sigma(t,T)dW(t)$$

Forward volatility satisfies the separability condition

$$\sigma(t,T) = g(t)h(T)$$
$$= \sigma\, e^{-\kappa(T-t)}$$

One can derive the instantaneous forward rate

$$F(t,T) = F(0,T)\exp\left\{e^{-\kappa(T-t)}X_i(t) - 0.5\big(V(0,T) - V(t,T)\big)\right\}$$

$$dX(t) = -\kappa X(t)dt + \sigma dW(t)$$

$$V(t,T) = \exp\left(\sigma^2 \frac{1-e^{-2\kappa T}}{2\kappa}\right)$$

Mean reverting single factor derives the entire forward curve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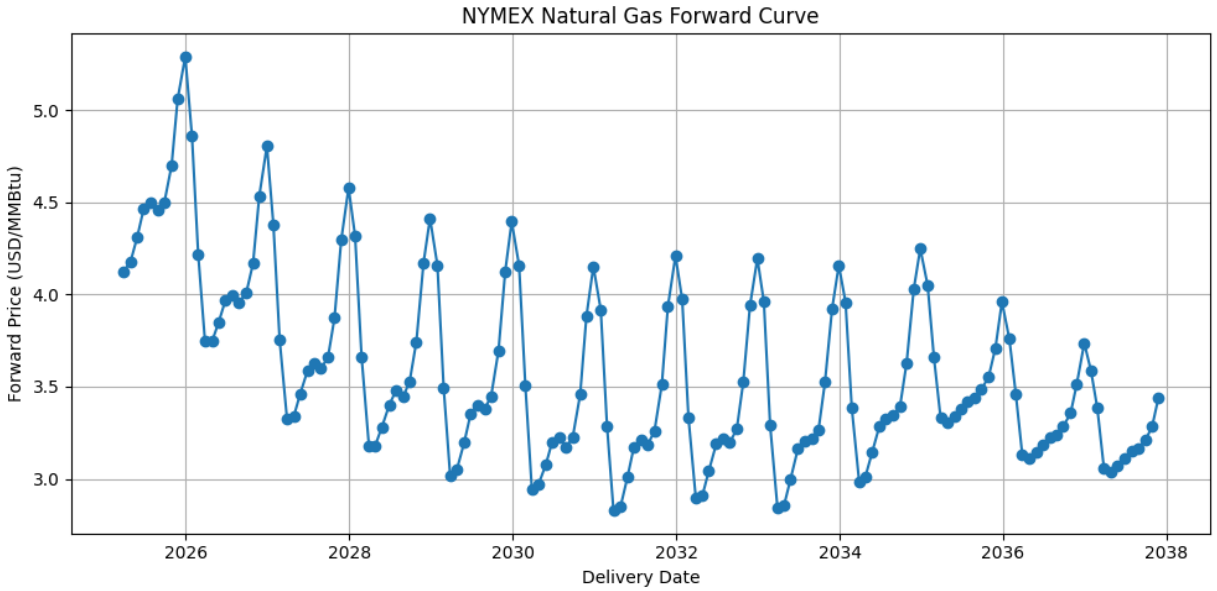
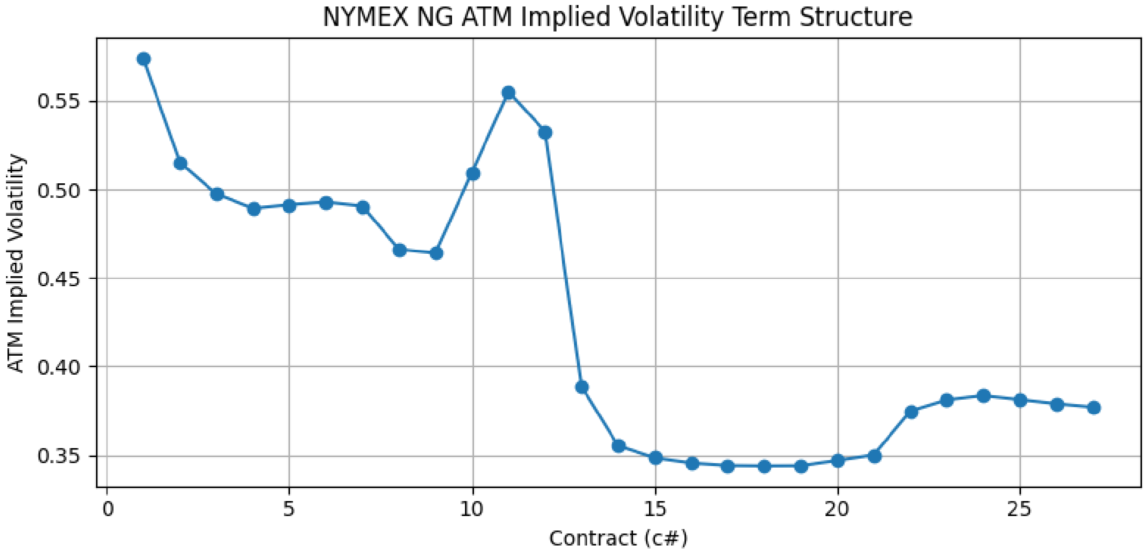
Seasonality

Seasonality is observed in the market for both commodity future price curves and option volatilities.





Incorporating Seasonality

Seasonality enters as a deterministic expiry-dependent scaling in the forward volatility

$$s(T) = \exp\{a(T)\}$$

$$\sigma(t,T) = s(T)\,\sigma\, e^{-\kappa(T-t)}$$

Future price curve takes the form

$$F(t,T) = F(0,T)\exp\left\{e^{s(T)-\kappa(T-t)}X(t) - 0.5\left(\tilde{V}(0,T) - \tilde{V}(t,T)\right)\right\}$$

$$dX(t) = -\kappa X(t)dt + \sigma dW(t)$$

$$\tilde{V}(t,T) = \exp\left(\sigma^2 e^{2s(T)}\frac{1-e^{-2\kappa T}}{2\kappa}\right)$$

European Option Prices

Consider a K-strike European call option on a T -maturity future, paying $(F(T',T) - K)^+$ at the option maturity T', where $T' \le T$ and $K > 0$. Then,

$$C(0) = P(0,T')\{F(0,T)\Phi\left(d_+(T',T)\right) - K\Phi\left(d_-(T',T)\right)\}$$

with

$$d_\pm(T',T) = \frac{\ln(F(0,T)/K) \pm 0.5\tilde{V}(T',T)^2}{\tilde{V}(T',T)}$$

Calibration Strategy: “FirstBestFitThenBootstrap”

- Set log-seasonality parameters a(T) to 0 for all T’s, i.e. s(T) =1.
- Calibrate σ and κ on ATM Commodity European option quotes.
- Bootstrap a(T)'s while keeping calibrated σ and κ fixed.

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Calibration Results

BestFit:

Calibration details:

#	time	modelVol	marketVol	(diff)	modelValue	marketValue	(diff)	Sigma	Kappa	Seasonality
0	0.0849315	0.502083	0.543324	−0.0412405	0.241092	0.260856	−0.0197633	0.505567	0.163184	1
1	0.167123	0.498766	0.503382	−0.00461508	0.340894	0.344049	−0.00315429	0.505567	0.163184	1
2	0.252055	0.495345	0.487593	0.00775213	0.42956	0.422871	0.00668867	0.505567	0.163184	1
3	0.334247	0.492088	0.480616	0.0114722	0.505627	0.493916	0.0117106	0.505567	0.163184	1
4	0.419178	0.488759	0.480855	0.00790377	0.564357	0.555305	0.00905163	0.505567	0.163184	1
5	0.50411	0.485468	0.479842	0.00562525	0.610967	0.603957	0.00701038	0.505567	0.163184	1
6	0.586301	0.48232	0.470957	0.0113634	0.664895	0.649404	0.015491	0.505567	0.163184	1
7	0.671233	0.479116	0.451792	0.027324	0.741636	0.699858	0.0417782	0.505567	0.163184	1
8	0.753425	0.476031	0.458165	0.0178659	0.836956	0.805978	0.0309781	0.505567	0.163184	1
9	0.838356	0.472877	0.489302	−0.0164253	0.894495	0.925058	−0.0305625	0.505567	0.163184	1
10	0.923288	0.469766	0.510487	−0.0407206	0.842986	0.914662	−0.071676	0.505567	0.163184	1
11	1	0.466988	0.473299	−0.0063108	0.760976	0.771071	−0.0100944	0.505567	0.163184	1

rmse = 0.0408763

FirstBestFitThenBootstrap:

Calibration details:

#	time	modelVol	marketVol	(diff)	modelValue	marketValue	(diff)	Sigma	Kappa	Seasonality
0	0.0849315	0.543324	0.543324	0	0.260856	0.260856	−2.22045e−16	0.505567	0.163184	1.08214
1	0.167123	0.503382	0.503382	0	0.344049	0.344049	0	0.505567	0.163184	1.00929
2	0.252055	0.487593	0.487593	0	0.422871	0.422871	−2.22045e−16	0.505567	0.163184	0.98435
3	0.334247	0.480616	0.480616	0	0.493916	0.493916	2.22045e−16	0.505567	0.163184	0.976688
4	0.419178	0.480855	0.480855	0	0.555305	0.555305	−2.22045e−16	0.505567	0.163184	0.98383
5	0.50411	0.479842	0.479842	0	0.603957	0.603957	0	0.505567	0.163184	0.988413
6	0.586301	0.470957	0.470957	0	0.649404	0.649404	0	0.505567	0.163184	0.976444
7	0.671233	0.451792	0.451792	0	0.699858	0.699858	0	0.505567	0.163184	0.943
8	0.753425	0.458165	0.458165	0	0.805978	0.805978	0	0.505567	0.163184	0.962486
9	0.838356	0.489302	0.489302	0	0.925058	0.925058	0	0.505567	0.163184	1.03473
10	0.923288	0.510487	0.510487	0	0.914662	0.914662	0	0.505567	0.163184	1.08668
11	1	0.473299	0.473299	0	0.771071	0.771071	2.22045e−16	0.505567	0.163184	1.01351

rmse = 3.47036e−16

General N-Factor Model

N-Factor Model is defined by following dynamics: Andersen[2008]: “Markov Models for Commodity Futures: Theory and Practice”

$$dF(t,T) = F(t,T)\mathbf{1}^{\mathbf{T}}\mathbf{S}(T)H(T)^T g(t)^T dW(t)$$

$$F(t,T) = F(0,T)\exp\left\{-\frac{1}{2}s(t,T)^T y(t)s(t,T) + s(t,T)^T x(t)\right\}$$

$$dx(t) = -\boldsymbol{\kappa}(t)\,x(t)dt + \boldsymbol{\Sigma}^T dW(t)$$

- $W(t)$ is a n-dimensional Q-Brownian motion, $(W_i)_{i=\{1,..,n\}}$
- $\boldsymbol{\kappa}$ is a n-dimensional vector, $(\kappa_i)_{i=\{1,..,n\}}$
- $\boldsymbol{\Sigma}$ is a $n \times n$ matrix, $\left[\sigma_{\mathrm{ij}}\right]_{i,j=\{1,..,n\}}$
- $\mathrm{S}(t)$ is a $n \times n$ diagonal matrix, $diag\left[e^{a_i(T)}\right]_{i=\{1,..,n\}}$
- $g(t)$ is a $n \times n$ matrix valued function, $\left[\sigma_{\mathrm{ij}} e^{-\int_0^t \kappa_i(u)du}\right]_{i,j=\{1,..,n\}}$
- $\mathrm{H}(t)$ is a $n \times n$ diagonal matrix, $diag\left[e^{-\int_0^t \kappa_i(u)du}\right]_{i=\{1,..,n\}}$
- $s(t,T)$ is a n-dimensional vector valued function. $\left(e^{s_1(T)} e^{-\int_t^T \kappa_{\mathrm{i}}(u)du}\right)_{i=\{1,..,n\}}$
- $y(t)$ is a $n \times n$ matrix valued function, $\int_0^t g(u)^T g(u)du$

Two-Factor Model

Andersen notes that two factors explain ~ 95% of curve movements (PCA in natural gas market) and hence chooses $\boldsymbol{\kappa}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{S}(T)$ as below

$$\boldsymbol{\kappa} = [\kappa \quad 0], \qquad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & \sigma_\infty \\ \sigma_2 & 0 \end{bmatrix}, \qquad \boldsymbol{S}(T) = \begin{bmatrix} e^{a_1(T)} & 0 \\ 0 & e^{a_2(T)} \end{bmatrix}$$

Then, state variable $x(t)$ can be rewritten as follows

$$dx_1(t) = -\kappa x_1(t)dt + \sigma_1 dW_1(t) + \sigma_2\, dW_2(t)$$
$$dx_2(t) = \sigma_\infty dW_1(t)$$

Future price curve has the following dynamics:

$$dF(t,T) = F(t,T)\left\{\left(e^{a_1(T)}e^{-\kappa(T-t)}\sigma_1 + e^{a_2(T)}\sigma_\infty\right)dW_1(t) + e^{a_1(T)}e^{-\kappa(T-t)}\sigma_2 dW_2(t)\right\}$$

Two factor Model: Interpretation

Then the instantaneous forward rate takes the form

$$dF(t,T) = F(t,T)e^{a_2(T)}\left\{\underbrace{\left(e^{d(T)}e^{-\kappa(T-t)}\sigma_1 + \sigma_\infty\right)}_{=:\ \sigma_1(t,T)} dW_1(t) + \underbrace{e^{d(T)}e^{-\kappa(T-t)}\sigma_2}_{=:\ \sigma_2(t,T)}\, dW_2(t)\right\}$$

where $d(T) = a_1(T) - a_2(T)$.

- $\sigma_2(t,T)$ primarily affects the short-end of the futures curve as it decays exponentially at a rate of κ as $T - t$ increases
- $\sigma_1(t,T)$ has a similar decaying term -i.e. affects the short-end as well-, but also contains a term σ_∞ that persists for long futures maturities.
- $a_2(T)$ controls level of term volatility wrt. contract expiry (i.e. seasonality effect on term volas)
- $d(T)\big(= a_1(T) - a_2(T)\big)$ controls correlation seasonality -how strongly different maturities co-move across the year-. If $d(T)$ is set to zero, correlation would be time-stationary.

$$\rho(t,\Delta_1,\Delta_2) = \frac{e^{d(T_1)}e^{d(T_2)}e^{-\kappa(\Delta_1+\Delta_2)} + q\left(e^{d(T_1)}e^{-\kappa\Delta_1} + e^{d(T_2)}e^{-\kappa\Delta_2}\right) + w}{\sqrt{e^{2d(T_1)}e^{-2\kappa\Delta_1} + 2qe^{d(T_1)}e^{-\kappa\Delta_1} + w}\sqrt{e^{2d(T_2)}e^{-2\kappa\Delta_2} + 2qe^{d(T_2)}e^{-\kappa\Delta_2} + w}};$$

- Parametrisations: $\sigma_{short} = \sqrt{(\sigma_1 + \sigma_\infty)^2 + \sigma_2^2}$, $\sigma_{long} = \sigma_\infty$, $\rho_\infty = \frac{\sigma_1 + \sigma_\infty}{\sigma_{short}}$

Calibration Strategy: "FirstBestFitThenBootstrap"

Model takes the initial term structure of futures prices as exogenously given quantities.

Model parameters: $[\, \sigma_{short}, \sigma_{long}, \ \rho_{\infty}, \kappa, \ a_1(T), \ a_2(T)]$

A possible calibration procedure:

1. Pick a value for ρ_{∞} based on empirical data (e.g. 0.5)
2. Temporarily set $a_1(T)$ and $a_2(T)$ to zero (remove seasonality).
3. Calibrate $\sigma_{short}, \sigma_{long}$ and κ that best fit the model option prices to ATM market option prices.
4. Set $a_1(T) = a_2(T)$, i.e. $d(T) = 0$.
5. Bootstrap $a_2(T)$ by matching the model to market ATM volatilities.


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Calibration Results

FirstBestFitThenBootstrap:

Calibration details:

#	time	modelVol	marketVol	(diff)	modelValue	marketValue	(diff)	SigmaShort	SigmaLong	Rho	Kappa	Seasonality
0	0.0849315	0.543324	0.543324	0	0.260856	0.260856	0	0.5883	0.484394	0.5	4.67163	1.01478
1	0.167123	0.503382	0.503382	0	0.344049	0.344049	0	0.5883	0.484394	0.5	4.67163	0.994166
2	0.252055	0.487593	0.487593	0	0.422871	0.422871	-2.22045e-16	0.5883	0.484394	0.5	4.67163	0.995289
3	0.334247	0.480616	0.480616	0	0.493916	0.493916	-4.996e-16	0.5883	0.484394	0.5	4.67163	0.998022
4	0.419178	0.480855	0.480855	0	0.555305	0.555305	0	0.5883	0.484394	0.5	4.67163	1.00745
5	0.50411	0.479842	0.479842	0	0.603957	0.603957	2.22045e-16	0.5883	0.484394	0.5	4.67163	1.00928
6	0.586301	0.470957	0.470957	0	0.649404	0.649404	-2.22045e-16	0.5883	0.484394	0.5	4.67163	0.991845
7	0.671233	0.451792	0.451792	0	0.699858	0.699858	-4.44089e-16	0.5883	0.484394	0.5	4.67163	0.951432
8	0.753425	0.458165	0.458165	0	0.805978	0.805978	-5.55112e-16	0.5883	0.484394	0.5	4.67163	0.964178
9	0.838356	0.489302	0.489302	0	0.925058	0.925058	0	0.5883	0.484394	0.5	4.67163	1.02866
10	0.923288	0.510487	0.510487	0	0.914662	0.914662	2.22045e-16	0.5883	0.484394	0.5	4.67163	1.07202
11	1	0.473299	0.473299	0	0.771071	0.771071	0	0.5883	0.484394	0.5	4.67163	0.992944

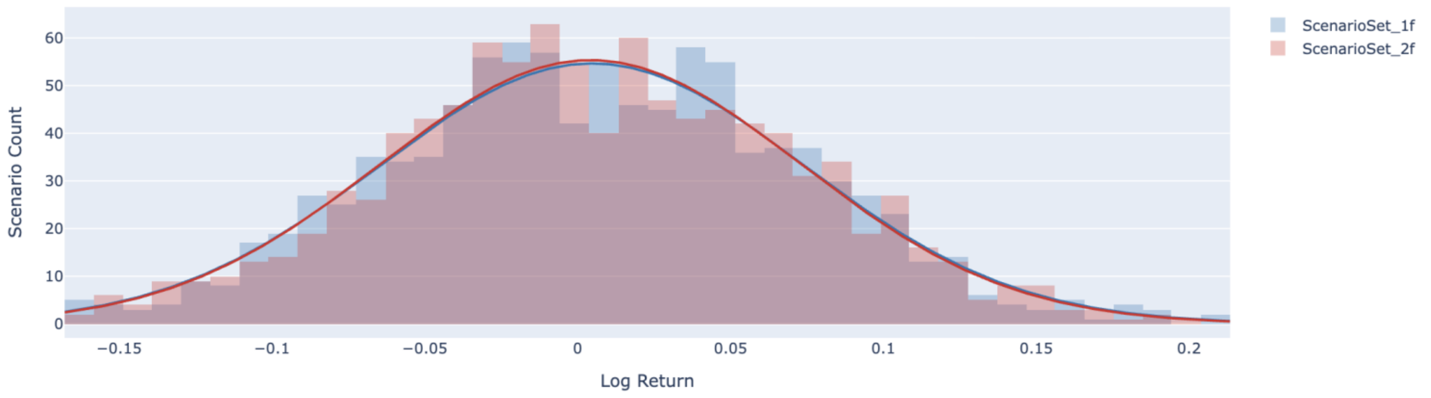
rmse = 4.55237e-16

CommodityCurve/NYMEX:NG — Log-Return Scenario Distributions

Date: 2026-04-27

Tenor: 0

Date: 2026-04-27 | Tenor 0 — ScenarioSet_1f: μ=0.00469, σ=0.06958, ϰ=0.25 | ScenarioSet_2f: μ=0.00425, σ=0.06870, ϰ=0.04



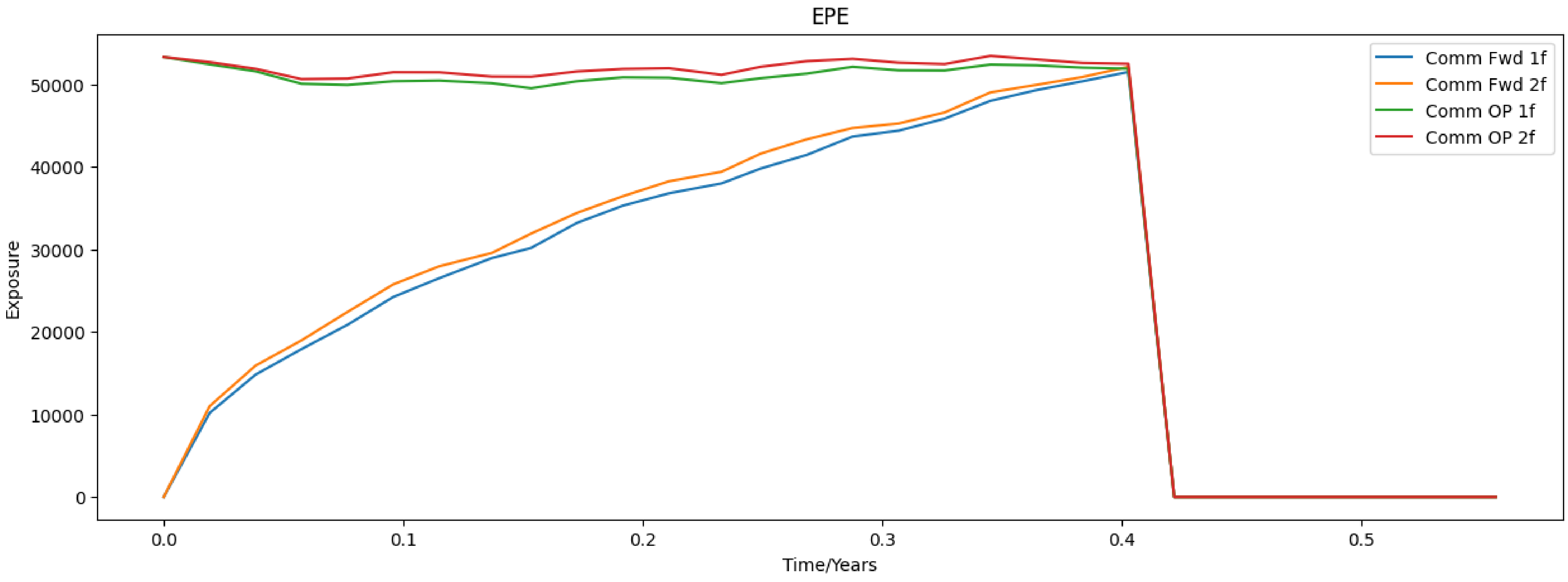
CommodityCurve/NYMEX:NG — Log-Return Scenario Distributions

Date: 2026-04-27

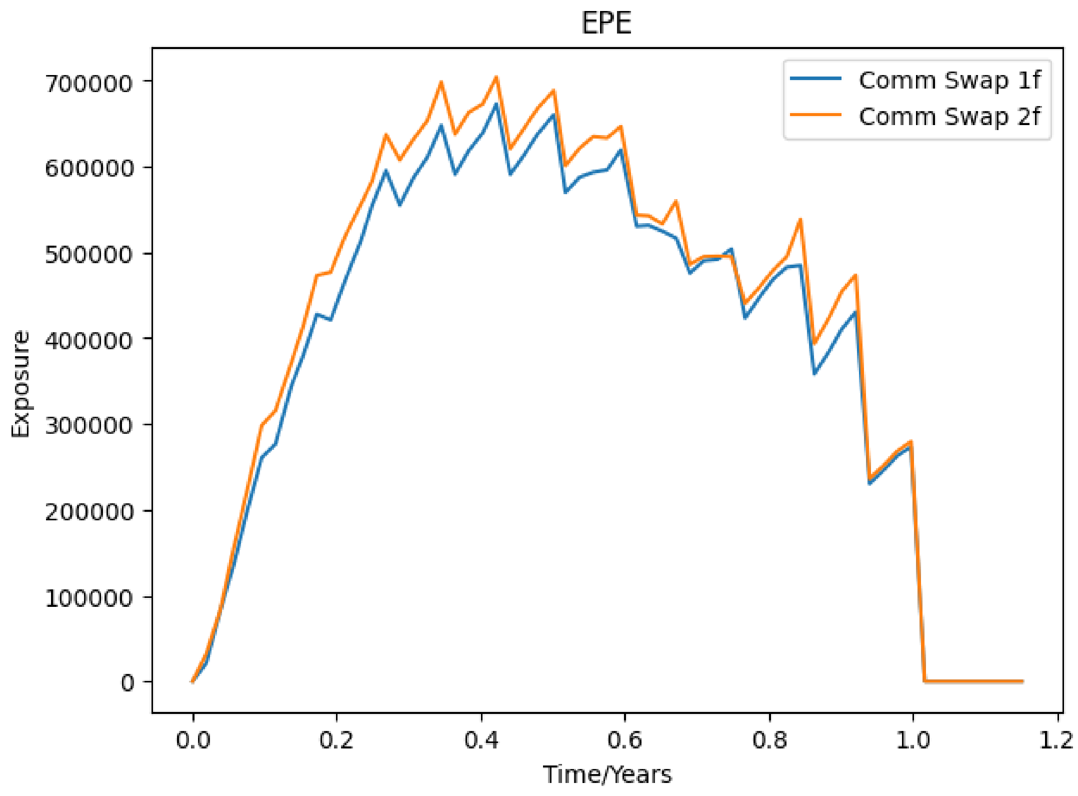
Tenor: 11

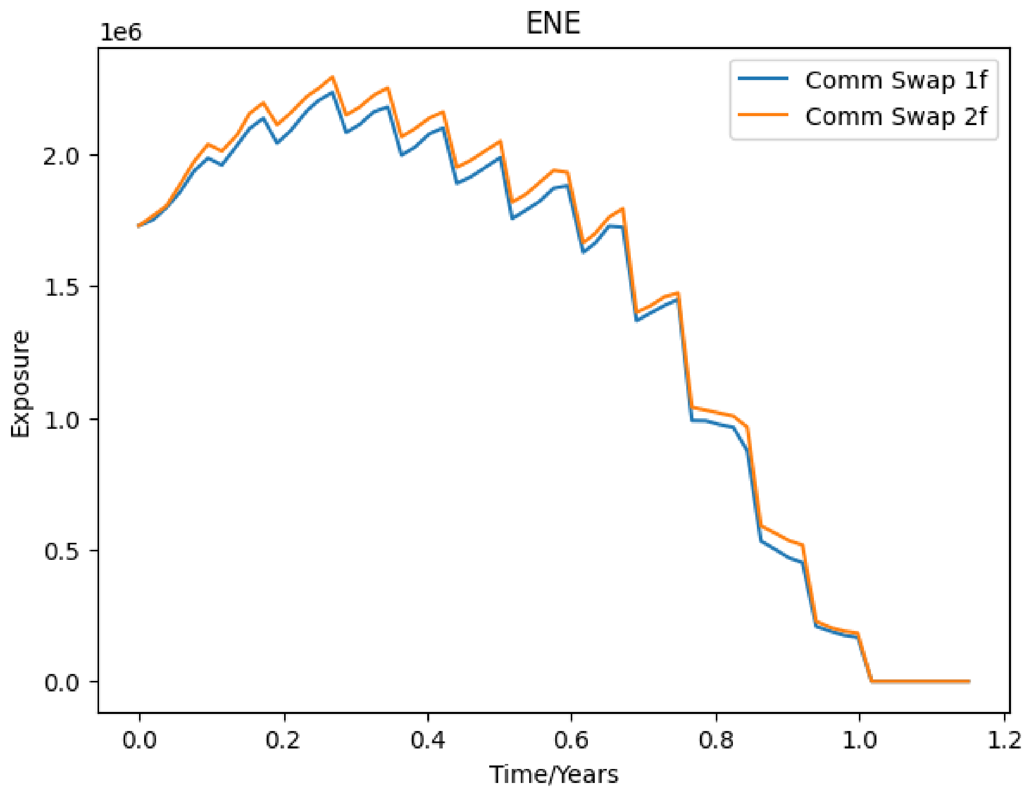
Date: 2026-04-27 | Tenor 11 — ScenarioSet_1f: μ=-0.00016, σ=0.05990, ϰ=0.25 | ScenarioSet_2f: μ=-0.00113, σ=0.06894, ϰ=-0.10

Exposure Profile: Commodity Option vs Forward



Exposure Profile: Commodity Swap





Take aways:

- Seasonality, introduced as an expiry-dependent scaling factor, improves calibration fit even within a one-factor framework**.**
- The two-factor model captures both short-end and long-end dynamics of the futures curve, resulting in more realistic distributions and exposure profiles.

Next steps:

- Relax time stationary correlation assumption ($a_1(T) = a_2(T)$) in the two-factor calibration
- Investigate exposure profiles of calendar spread options
- Calibration of higher factor models

Appendix

N-Factor Model

LSEG

Multi factor model

Instantaneous forward rare $F(t,T)$is modeled in multi-factor HJM frame-work

$$dF(t,T) = F(t,T)\sigma(t,T)^T dW(t)$$

where

- $W(t)$ is a n-dimensional Q-Brownian motion
- $\sigma(t,T)$ is a n-dimensional F_t-adapted process

The separability condition

$$\sigma(t,T) = g(t)h(T)$$

- $h(t)$ is a n-dimensional vector valued function given by

$$h_i(t) = e^{-\int_0^t \kappa_i(u)du}$$

- $g(t)$ is a $n \times n$ matrix valued function

$$g(t) = \Sigma\, H(t)^{-1}$$

Multi factor model cont'd

$$H(t) \coloneqq diag\big(h_1(t), \dots, h_n(t)\big) \text{ and } \Sigma \coloneqq \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{nn} \end{bmatrix}$$

$$g(T) \coloneqq \begin{bmatrix} \sigma_{11} e^{-\int_0^T \kappa_1(u)du} & \sigma_{12} e^{-\int_0^T \kappa_2(u)du} & \cdots & \sigma_{1n} e^{-\int_0^T \kappa_n(u)du} \\ \sigma_{21} e^{-\int_0^T \kappa_1(u)du} & \sigma_{22} e^{-\int_0^T \kappa_2(u)du} & \cdots & \sigma_{2n} e^{-\int_0^T \kappa_n(u)du} \\ \vdots & \vdots & \ddots & \vdots\, e^{-\int_0^T \kappa_n(u)du} \\ \sigma_{n1} e^{-\int_0^T \kappa_1(u)du} & \sigma_{n2} e^{-\int_0^T \kappa_2(u)du} & \cdots & \sigma_{nn} e^{-\int_0^T \kappa_n(u)du} \end{bmatrix}$$

One can derive the instantaneous forward rate formula as follows

$$F(t,T) = F(0,T)\exp\left\{-\frac{1}{2}h(T)^T y(t) h(T) + h(T)^T z(t)\right\}.$$

where $z(t)$ is given by the following SDE

$$dz(t) = g(t)^T dW(t)$$

$$y(t) = \int_0^t g(u)^T g(u)du$$

Multi factor model: Seasonality

Seasonality adjustment for the i^{th} factor is denoted by $s_i(T)$ and defined as follows:

$$s_i(T) \coloneqq e^{a_i(T)}$$

Then, $n \times n$ diagonal matrix $S(T)$defines the seasonality

$$S(T) \coloneqq diag\left(s_1(T), \dots, s_n(T)\right)$$

After lengthy calculations

$$F(t,T) = F(0,T)\exp\{-\frac{1}{2}\underbrace{\mathbf{1^T S}(T)H(T)H(t)^{-1}}_{s(t,T)^T}y(t)\underbrace{H(t)^{-1}H(T)S(T)}_{s(t,T)} + \mathbf{1^T}S(T)H(T)z(t)\}$$

$$= F(0,T)\exp\left\{-\frac{1}{2}s(t,T)^T y(t)s(t,T) + \mathbf{1^T}S(T)H(T)z(t)\right\}$$

Multi factor model: Mean reverting process

In practical applications, the volatility of $x(t)$ will often exhibit exponential growth as a function of t, a feature that may complicate numerical work. It is therefore often useful to change variables such that the primary stochastic driver is mean-reverting

$$x(t) \coloneqq H(t)z(t)$$

Then, from product rule follows

$$dx(t) = -\kappa(t)x(t)dt + \sigma_x(t)^T dW(t)$$

Future price can be rewritten as follows:

$$F(t,T) = F(0,T)\exp\left\{-\frac{1}{2}s(t,T)^T y(t)s(t,T) + s(t,T)^T x(t)\right\}$$

Multi factor model: European option pricing

Consider a K-strike European call option on a T-maturity future, paying $(F(T',T)-K)^+$at the option maturity T',where $T' \leq T$ and $K > 0$. Then the time 0 arbitrage-free value of the call option is

$$C(0,T',T,K) = P(0,T')\left\{F(0,T)\Phi\big(d_+(T',T)\big) - K\Phi\big(d_-(T',T)\big)\right\}$$

with

$$d_\pm(T',T) = \frac{\ln(F(0,T)/K) \pm 0.5v(T',T)^2}{v(T',T)}$$
$$v(T',T) = s(T',T)^T y(T')s(T',T)$$

Update on User Interfaces:

SWIG

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Quantitative Consultant

Eric Ehlers

Principal Quantitative Consultant

Post Trade Solutions

Risk Analytics Lab Demo

Post Trade Solutions

Jake Ullman

Senior Business Development Associate